

# Nuclear matter with a Bose condensate of dibaryons in relativistic Hartree approximation

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## Abstract

The Green's functions are constructed and a one-loop calculation is given for the scalar and vector densities and for the equation of state of nuclear matter with a Bose condensate of dibaryons. This is the lowest approximation in the loop expansion of quantum hadrodynamics, sufficient to account for the presence of dibaryons not in the condensate in the heterophase nucleon-dibaryon matter. It leads to a finite effective nucleon mass and remains consistent with increasing the density.

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# 1 Introduction

The prospect of observing the long-lived H-particle predicted in 1977 by Jaffe [1] stimulated considerable activities in the experimental search for dibaryons [2]. Although no positive evidence has been observed yet, future experimental studies may elucidate the nature of this interesting possibility. Non-strange dibaryons which have a small width due to zero coupling to the  $NN$ -channel are promising candidates for experimental searches [3]. A method for searching exotic dibaryon resonances in the double proton-proton bremsstrahlung reaction is discussed in Ref. [4]. The narrow peaks observed by Bilger *et al.* [5] and Clement *et al.* [6] in the pion double charge exchange (DCE) reactions on nuclei have been interpreted by Martemyanov and Schepkin [7] as evidence for the existence of a narrow  $d'$  dibaryon with a mass of 2060 MeV. Recent experiments at TRIUMF (Vancouver) and CELSIUS (Uppsala) seem to support the existence of the  $d'$  [8]. Recently, some indications for a  $d_1(1920)$  dibaryon have also been found [9].

The existence of dibaryons when settled reliably by experiments should have important implications for the properties of nuclear matter. The chemical potential of nucleons increases with the density. When it becomes higher than half of the chemical potential of dibaryons, the production of dibaryons becomes energetically favourable. Dibaryons are Bose particles. In nuclear matter they form a Bose condensate. This interesting phenomenon is studied by Baldin *et al.* [10] and Chizov *et al.* [11] by including dibaryon interactions through a van der Waals volume correction. A model for nuclear matter with an admixture of dibaryons with the short-range nuclear forces approximated by a  $\delta$ -function-like pseudopotential is discussed in Ref. [12]. The relativistic mean-field theory (MFT) with dibaryons was analysed recently by Faessler *et al.* [13]-[15]. An exactly solvable one-dimensional model for the dibaryon condensation has been found by Buchmann *et al.* [16]. Heterophase nucleon-dibaryon matter (HNDM) can be formed in interiors of neutron stars [17]-[19] and in heavy-ion collisions [15]. The physics of heterophase substances is reviewed in Ref. [20].

In previous works [10]-[15], [17]-[20], it was assumed that at zero temperature dibaryons in nuclear matter are all in the condensate. In Bogoliubov's model of a dilute interacting Bose gas [21], a fraction of Bose particles is out of the condensate [22, 23]. This fraction increases with the density and has an important effect on various thermodynamical quantities.

MFT constitutes the basis of quantum hadrodynamics (QHD) [24]-[26]. In a loop expansion of QHD, it corresponds to the lowest order term (no loops). In MFT all dibaryons are in the condensate. Dibaryons that are not in the condensate appear first in a one-loop calculation of the scalar and vector densities and of the equation of state

(EOS). This approximation is identical to the relativistic Hartree approximation (RHA) for normal nuclear matter. It is sufficient to account for the existence of non-condensate dibaryons. In this paper, we construct Green's functions of HNMD and give a one-loop calculation of the scalar and vector densities and the EOS of the binary nucleon-dibaryon mixture.

## 2 Quantum hadrodynamics with dibaryons

The dibaryonic extension of QHD is determined by the Lagrangian density

$$\begin{aligned} \mathcal{L} = & \bar{\Psi}(i\partial_\mu\gamma_\mu - m_N - g_\sigma\sigma - g_\omega\omega_\mu\gamma_\mu)\Psi + \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}F_{\mu\nu}^2 \\ & + \frac{1}{2}m_\omega^2\omega_\mu^2 - \frac{1}{2}\lambda(\partial_\mu\omega_\mu)^2 + (\partial_\mu - ih_\omega\omega_\mu)\varphi^*(\partial_\mu + ih_\omega\omega_\mu)\varphi - (m_D + h_\sigma\sigma)^2\varphi^*\varphi. \end{aligned} \quad (1)$$

Here,  $\Psi$  is the nucleon field,  $\omega_\mu$  and  $\sigma$  are fields of the  $\omega$ - and  $\sigma$ -mesons,  $F_{\mu\nu} = \partial_\nu\omega_\mu - \partial_\mu\omega_\nu$  is the field strength tensor, and  $\varphi$  is the dibaryon isoscalar-scalar (or isoscalar-pseudoscalar) field. Furthermore,  $m_\omega$  and  $m_\sigma$  are the  $\omega$ - and  $\sigma$ -meson masses and  $g_\omega$ ,  $g_\sigma$ ,  $h_\omega$ ,  $h_\sigma$  are coupling constants of the  $\omega$ - and  $\sigma$ -mesons with nucleons ( $g$ ) and dibaryons ( $h$ ). The  $\omega$ -meson field is described by Stueckelberg's equation [27]. The limit  $\lambda \rightarrow 0$  recovers the Proca equation. The MFT results [13]-[15] do not depend on the value of  $\lambda$  in Eq.(1).

The field operators can be expanded in  $c$ -numbers and operator parts:  $\omega_\mu = g_{\mu 0}\omega_c + \hat{\omega}_\mu$ ,  $\sigma = \sigma_c + \hat{\sigma}$ ,  $\varphi = \varphi_c + \hat{\varphi}$ , and  $\varphi^* = \varphi_c^* + \hat{\varphi}^*$ . The  $\sigma$ -meson mean field determines the effective nucleon and dibaryon masses in the medium:

$$\begin{aligned} m_N^* &= m_N + g_\sigma\sigma_c, \\ m_D^* &= m_D + h_\sigma\sigma_c. \end{aligned} \quad (2)$$

The nucleon scalar and vector densities are defined by the expectation values

$$\rho_{NS} = \langle \bar{\Psi}\Psi \rangle, \quad (3)$$

$$\rho_{NV} = \langle \bar{\Psi}\gamma_0\Psi \rangle. \quad (4)$$

The chemical potential equals  $\mu_N = \mu_N^* + g_\omega\omega_c$  and  $\mu_N = -\mu_N^* + g_\omega\omega_c$ , respectively, for nucleons and for nucleon holes in the Dirac sea. Here,  $\mu_N^* = +\sqrt{m_N^{*2} + k_F^2}$  is the nucleon Fermi energy and  $k_F$  is the Fermi wave number.

The dibaryon scalar density is defined as an additional source of the  $\sigma$ -meson mean field

$$2m_D^*\rho_{DS} = \langle 2(m_D + h_\sigma\sigma)\varphi^*\varphi \rangle. \quad (5)$$

The condensate dibaryons give a contribution  $\rho_{DS}^c = |\varphi_c|^2$  to the scalar density. The time evolution of the condensate field  $\varphi_c$  is determined by the chemical potential  $\mu_D$ :

$$\varphi_c(t) = e^{-i\mu_D t} \sqrt{\rho_{DS}^c}. \quad (6)$$

To eliminate the time dependence from the condensate parts of the  $\varphi$  fields, we pass to the  $\mu$ -representation for dibaryons. This can be done by the substitution  $\varphi \rightarrow \varphi e^{i\mu_D t}$  and  $\varphi^* \rightarrow \varphi^* e^{-i\mu_D t}$ . The dibaryon vector density is equal to the timelike component of the expectation value of the dibaryon current

$$\langle j_\mu^D \rangle = \langle \varphi^* (2\mu_D - 2h_\omega \omega + i \overleftrightarrow{\partial})_\mu \varphi \rangle. \quad (7)$$

Here,  $(\mu_D)_\mu = (\mu_D, \mathbf{0})$  in the rest frame of the substance. The condensate dibaryons give a contribution  $\rho_{DV}^c = 2\mu_D^* \rho_{DS}^c$  to the vector density with  $\mu_D = \mu_D^* + h_\omega \omega_c$ .

The self-consistency equation for the effective nucleon mass is:

$$m_N^* = m_N - \frac{g_\sigma}{m_\sigma^2} (g_\sigma \rho_{NS} + h_\sigma 2m_D^* \rho_{DS}). \quad (8)$$

The  $\omega$ -meson mean field is determined from the equation

$$m_\omega^2 \omega_c = g_\omega \rho_{NV} + h_\omega \rho_{DV}.$$

### 3 Green's functions of heterophase nucleon-dibaryon matter

The model of Eq.(1) is analysed within MFT at zero temperature in Refs. [13, 14]. To go beyond MFT, it is necessary to construct Green's functions of the bosons

$$iD^{AB}(x' - x) = \langle T \hat{A}(x') \hat{B}(x) \rangle \quad (9)$$

with  $A, B = \sigma, \omega_\mu, \varphi, \varphi^*$ . For a one-loop calculation, it is sufficient to construct the Green's functions in the no-loop approximation. Above the critical density for formation of dibaryons, the  $\sigma$ - and  $\omega$ -mesons can be absorbed by the dibaryons in the condensate. As a result, the dibaryons leave the condensate and propagate as normal particles. These processes occur at tree level, and the Green's functions in the no-loop approximation describe the effect of  $\sigma - \omega - \varphi - \varphi^*$  mixing.

The Green's functions can be determined self-consistently by solving a system of Gorkov-Dyson equations. Multiplying the equation of motion for the  $\sigma$  field corresponding to the Lagrangian density (1) by  $\hat{\sigma}$  and taking the time-ordered product, we find the average value of the equation over the ground state

$$(-\square - m_\sigma^2) < T\sigma(1)\hat{\sigma}(2) > = \delta^4(1, 2) + g_\sigma < T\bar{\Psi}(1)\Psi(1)\hat{\sigma}(2) > \\ + 2h_\sigma < T(m_D + h_\sigma\sigma(1))\varphi^*(1)\varphi(1)\hat{\sigma}(2) > .$$

Taking into account the second-order terms with respect to the operator fields (one-loop approximation), one gets

$$(-\square - m_\sigma^2) < T\hat{\sigma}(1)\hat{\sigma}(2) > = \delta^4(1, 2) + 2h_\sigma^2\rho_{DS}^c < T\hat{\sigma}(1)\hat{\sigma}(2) > \\ + 2m_D^*h_\sigma\sqrt{\rho_{DS}^c}(< T\hat{\varphi}(1)\hat{\sigma}(2) > + < T\hat{\varphi}^*(1)\hat{\sigma}(2) >).$$

In the momentum representation, the equation takes the form

$$D^{\sigma\sigma}(k) = \tilde{D}^{\sigma\sigma}(k) + \tilde{D}^{\sigma\sigma}(k)2m_D^*h_\sigma\sqrt{\rho_{DS}^c}(D^{\varphi\sigma}(k) + D^{\varphi^*\sigma}(k)). \quad (10)$$

The equations for the other Green's functions can be obtained in similar way:

$$D_\mu^{\sigma\omega}(k) = \tilde{D}^{\sigma\sigma}(k)2m_D^*h_\sigma\sqrt{\rho_{DS}^c}(D_\mu^{\varphi\omega}(k) + D_\mu^{\varphi^*\omega}(k)), \quad (11)$$

$$D^{\sigma\varphi}(k) = \tilde{D}^{\sigma\sigma}(k)2m_D^*h_\sigma\sqrt{\rho_{DS}^c}(D^{\varphi^*\varphi}(k) + D^{\varphi\varphi}(k)), \quad (12)$$

$$D^{\sigma\varphi^*}(k) = \tilde{D}^{\sigma\sigma}(k)2m_D^*h_\sigma\sqrt{\rho_{DS}^c}(D^{\varphi^*\varphi^*}(k) + D^{\varphi\varphi^*}(k)), \quad (13)$$

$$D_{\mu\nu}^{\omega\omega}(k) = \tilde{D}_{\mu\nu}^{\omega\omega}(k) + \tilde{D}_{\mu\tau}^{\omega\omega}(k)h_\omega\sqrt{\rho_{DS}^c}[(2\mu_D^* + k)_\tau D_\nu^{\varphi\omega}(k) + (2\mu_D^* - k)_\tau D_\nu^{\varphi^*\omega}(k)], \quad (14)$$

$$D_\mu^{\omega\varphi}(k) = \tilde{D}_{\mu\tau}^{\omega\omega}(k)h_\omega\sqrt{\rho_{DS}^c}[(2\mu_D^* + k)_\tau D^{\varphi\varphi}(k) + (2\mu_D^* - k)_\tau D^{\varphi^*\varphi}(k)], \quad (15)$$

$$D_\mu^{\omega\varphi^*}(k) = \tilde{D}_{\mu\tau}^{\omega\omega}(k)h_\omega\sqrt{\rho_{DS}^c}[(2\mu_D^* + k)_\tau D^{\varphi\varphi^*}(k) + (2\mu_D^* - k)_\tau D^{\varphi^*\varphi^*}(k)], \quad (16)$$

$$D^{\varphi\varphi^*}(k) = \tilde{D}^{\varphi\varphi^*}(k) + \tilde{D}^{\varphi\varphi^*}(k)[h_\omega\sqrt{\rho_{DS}^c}(2\mu_D^* + k)_\tau D_\tau^{\omega\varphi^*}(k) + 2m_D^*h_\sigma\sqrt{\rho_{DS}^c}D^{\sigma\varphi^*}(k)], \quad (17)$$

$$D^{\varphi\varphi^*}(k) = \tilde{D}^{\varphi\varphi^*}(k)[h_\omega\sqrt{\rho_{DS}^c}(2\mu_D^* + k)_\tau D_\tau^{\omega\varphi^*}(k) + 2m_D^*h_\sigma\sqrt{\rho_{DS}^c}D^{\sigma\varphi}(k)], \quad (18)$$

$$D^{\varphi^*\varphi^*}(k) = \tilde{D}^{\varphi^*\varphi^*}(k)[h_\omega\sqrt{\rho_{DS}^c}(2\mu_D^* - k)_\tau D_\tau^{\omega\varphi^*}(k) + 2m_D^*h_\sigma\sqrt{\rho_{DS}^c}D^{\sigma\varphi^*}(k)]. \quad (19)$$

Here,

$$\tilde{D}^{\sigma\sigma}(k) = \frac{1}{k^2 - \tilde{m}_\sigma^2}, \\ \tilde{D}_{\mu\nu}^{\omega\omega}(k) = \frac{-g_{\mu\nu} + k_\mu k_\nu / \tilde{m}_\omega^2}{k^2 - \tilde{m}_\omega^2} - \frac{k_\mu k_\nu / \tilde{m}_\omega^2}{k^2 - \tilde{m}^2}, \\ \tilde{D}^{\varphi\varphi^*}(k) = \frac{1}{(k + \mu_D^*)^2 - m_D^{*2}}$$

are the  $\sigma$ - and  $\omega$ -meson and the dibaryon MFT propagators. The effective  $\sigma$ - and  $\omega$ -meson masses are

$$\tilde{m}_\sigma^2 = m_\sigma^2 + 2h_\sigma^2\rho_{DS}^c, \\ \tilde{m}_\omega^2 = m_\omega^2 + 2h_\omega^2\rho_{DS}^c. \quad (20)$$

The mass of the effective scalar in Stueckelberg's equation is given by  $\tilde{m}^2 = \tilde{m}_\omega^2/\lambda$ .

Equations (10) - (19) constitute a closed system of equations, which allows us to determine selfconsistently the boson propagators including  $\omega - \sigma - \varphi^* - \varphi$  mixing. For more details see Ref. [14].

The scalar and vector dibaryon densities are expressed in terms of the dibaryon Green's functions

$$\begin{aligned} D^{\varphi\varphi^*}(k) &= (\tilde{D}^{\varphi^*\varphi}(k)^{-1} - \Sigma^{\varphi^*\varphi}(k))/\Xi(k), \\ D^{\varphi\varphi}(k) &= \Sigma^{\varphi\varphi}(k)/\Xi(k), \\ D^{\varphi^*\varphi^*}(k) &= \Sigma^{\varphi^*\varphi^*}(k)/\Xi(k). \end{aligned} \quad (21)$$

The denominator  $\Xi(k)$  and the self-energy operators are given by the expressions

$$\begin{aligned} \Xi(k) &= (\tilde{D}^{\varphi\varphi^*}(k)^{-1} - \Sigma^{\varphi\varphi^*}(k))(\tilde{D}^{\varphi^*\varphi}(k)^{-1} - \Sigma^{\varphi^*\varphi}(k)) - \Sigma^{\varphi^*\varphi^*}(k)\Sigma^{\varphi\varphi}(k), \\ \Sigma^{\varphi\varphi^*}(k) &= \Sigma^{\varphi^*\varphi}(-k) = (h_\omega\sqrt{\rho_{DS}^c})^2(2\mu_D^* + k)_\mu \tilde{D}_{\mu\nu}^{\omega\omega}(k)(2\mu_D^* + k)_\nu \\ &\quad + (2m_D^*h_\sigma\sqrt{\rho_{DS}^c})^2\tilde{D}^{\sigma\sigma}(k), \\ \Sigma^{\varphi\varphi}(k) &= \Sigma^{\varphi^*\varphi^*}(-k) = (h_\omega\sqrt{\rho_{DS}^c})^2(2\mu_D^* + k)_\mu \tilde{D}_{\mu\nu}^{\omega\omega}(k)(2\mu_D^* - k)_\nu \\ &\quad + (2m_D^*h_\sigma\sqrt{\rho_{DS}^c})^2\tilde{D}^{\sigma\sigma}(k). \end{aligned} \quad (22)$$

The structure of the dibaryon Green's functions is identical to the structure of the Green's functions of fermions in superconductors [23, 28].

From Eqs.(22) we see that the longitudinal component of the  $\omega$ -meson propagator contributes to the self-energy operators of the dibaryon Green's functions. The model is renormalizable for any finite value of  $\lambda$ . In the limit  $\lambda \rightarrow 0$  the finite renormalizable expressions become infinite. Now, we put  $\lambda = 1$  in order to obtain the  $\omega$ -meson propagator used in usual nuclear matter QHD calculations[25, 26]. In this case, the QHD results for ordinary nuclear matter are reproduced at low densities where there is no dibaryon condensate.

The chemical potential of dibaryons is determined from the relativistic extension of the Hugenholtz-Pines relation [29]

$$\mu_D^{*2} - m_D^{*2} = \Sigma^{\varphi\varphi^*}(0) - \Sigma^{\varphi\varphi}(0). \quad (23)$$

A relation of such a kind is necessary in order to get a pole in the dibaryon Green's functions at  $\omega = \mathbf{k} = 0$  and to guarantee the existence of sound in the medium. Eq.(23) gives

$$\mu_D^* = m_D^* \quad (24)$$

in agreement with MFT [13, 14].

## 4 One-loop scalar and vector densities and the EOS

The one-loop expression for the nucleon scalar density is calculated from Eq.(3). After renormalization, one gets [25, 26]

$$\rho_{NS} = \rho_{NS}^c + \rho'_{NS} \quad (25)$$

with

$$\rho_{NS}^c = \gamma_N \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{m_N^*}{\sqrt{m_N^{*2} + \mathbf{k}^2}} \theta(k_F - |\mathbf{k}|), \quad (26)$$

$$\rho'_{NS} = -4m_N^3 \zeta(m_N^*/m_N). \quad (27)$$

Here,  $\gamma_N$  is the statistical factor ( $\gamma_N = 2$  in neutron matter and  $\gamma_N = 4$  in nuclear matter) and

$$4\pi^2 \zeta(x) = x^3 \ln x + 1 - x - \frac{5}{2}(1-x)^2 + \frac{11}{6}(1-x)^3.$$

The one-loop expression for the dibaryon scalar density defined by Eq.(5) can be written in the form

$$2m_D^* \rho_{DS} = 2m_D^* \rho_{DS}^c + 2m_D^* \langle \hat{\varphi}^* \hat{\varphi} \rangle + 2h_\sigma \sqrt{\rho_{DS}^c} (\langle \hat{\sigma} \hat{\varphi} \rangle + \langle \hat{\sigma} \hat{\varphi}^* \rangle). \quad (28)$$

We extract the zero order contribution with respect to the dibaryon condensate:

$$\rho'_{DS} = i \int \frac{d^4 k}{(2\pi)^4} \tilde{D}^{\varphi\varphi^*}(k). \quad (29)$$

After renormalization, we obtain

$$2m_D^* \rho'_{DS} = m_D^3 \zeta(m_D^*/m_D). \quad (30)$$

The total dibaryon scalar density has the form

$$\rho_{DS} = \rho_{DS}^c + \rho'_{DS} + \rho''_{DS} \quad (31)$$

with

$$\rho''_{DS} = i \int \frac{d^4 k}{(2\pi)^4} \{ D^{\varphi\varphi^*}(k) - \tilde{D}^{\varphi\varphi^*}(k) + 4(h_\sigma \sqrt{\rho_{DS}^c})^2 \tilde{D}^{\sigma\sigma}(k) (D^{\varphi\varphi^*}(k) + D^{\varphi\varphi}(k)) \}. \quad (32)$$

This integral diverges. The renormalized expression is given by

$$2m_D^* \rho''_{DS} = \frac{1}{8\pi^2} \int_0^{+\infty} dk_E k_E^3 \lambda_{DS}^{ren}(k_E, m_D^*, \rho_{DS}^c) \quad (33)$$

where

$$\begin{aligned} \lambda_{DS}^{ren}(k_E, m_D^*, \rho_{DS}^c) &= \lambda_{DS}(k_E, m_D^*, \rho_{DS}^c) - \lambda_{DS}(k_E, m_D^*, 0) \\ &\quad - \rho_{DS}^c \frac{\partial \lambda_{DS}(k_E, m_D, 0)}{\partial \rho_{DS}^c} - (m_D^* - m_D) \rho_{DS}^c \frac{\partial^2 \lambda_{DS}(k_E, m_D, 0)}{\partial m_D^* \partial \rho_{DS}^c}. \end{aligned} \quad (34)$$

We have passed here into the Euclidean space. The density  $\lambda_{DS}(k_E, m_D^*, \rho_{DS}^c)$  has the form

$$\lambda_{DS}(k_E, m_D^*, \rho_{DS}^c) = \frac{4(k_E^2 E_\omega + m_D^{*2})z^2 + k_E^2(4E_\omega - 3 - k_E^2/m_D^{*2})}{2m_D^* E_\omega k_E^2 (\sqrt{z^2(z^2 + 1)} + z^2)} \quad (35)$$

where

$$\begin{aligned} E_\omega &= \frac{k_E^2 + m_\omega^2}{k_E^2 + \tilde{m}_\omega^2}, \\ z^2 &= \frac{k_E^2 + 8m_D^{*2}R}{4m_D^{*2}E_\omega}, \\ R &= \rho_{DS}^c \left( \frac{h_\omega^2}{k_E^2 + \tilde{m}_\omega^2} - \frac{h_\sigma^2}{k_E^2 + \tilde{m}_\sigma^2} \right). \end{aligned}$$

The counter terms in the Lagrangian density, responsible for the renormalization of the scalar density, are of the form  $\delta\mathcal{L}_s = (C_1\sigma + C_2\sigma^2)\varphi^*\varphi$ .

The one-loop expression for the dibaryon vector current has the form

$$\langle j_\mu^D \rangle = 2(\mu_D^*)_ \mu \rho_{DS}^c + \langle \hat{\varphi}^*(2\mu_D^* + i \overleftrightarrow{\partial})_\mu \hat{\varphi} \rangle - 2h_\omega \sqrt{\rho_{DS}^c} (\langle \hat{\omega}_\mu \hat{\varphi}^* \rangle + \langle \hat{\omega}_\mu \hat{\varphi} \rangle). \quad (36)$$

In the rest frame of the substance, the vector  $(\mu_D^*)_ \mu$  is defined by  $(\mu_D^*)_ \mu = (\mu_D^*, \mathbf{0})$ .

The nucleon vector density in RHA is the same as in MFT. The dibaryon vector density can be represented in the form

$$\rho_{DV} = \rho_{DV}^c + \rho'_{DV} + \rho''_{DV} \quad (37)$$

with

$$\rho'_{DV} = i \int \frac{d^4k}{(2\pi)^4} 2(\mu_D^* + \omega) \tilde{D}^{\varphi\varphi^*}(k) \equiv 0, \quad (38)$$

and

$$\begin{aligned} \rho''_{DV} &= i \int \frac{d^4k}{(2\pi)^4} \{ 2(\mu_D^* + \omega) (D^{\varphi\varphi^*}(k) - \tilde{D}^{\varphi\varphi^*}(k)) \\ &+ 4(h_\omega \sqrt{\rho_{DS}^c})^2 \tilde{D}^{\omega\omega}(k) (2\mu_D^* + \omega) (D^{\varphi\varphi^*}(k) + D^{\varphi\varphi}(k)) \}. \end{aligned} \quad (39)$$

In this expression,  $\omega$  is the timelike component of the vector  $k_\mu$ .

The renormalized expression for the value  $\rho''_{DV}$  is given by

$$\rho''_{DV} = \frac{1}{8\pi^2} \int_0^{+\infty} dk_E k_E^3 \lambda_{DV}^{ren}(k_E, m_D^*, \rho_{DV}^c) \quad (40)$$

with

$$\lambda_{DV}^{ren}(k_E, m_D^*, \rho_{DV}^c) = \lambda_{DV}(k_E, m_D^*, \rho_{DV}^c) - \lambda_{DV}(k_E, m_D^*, 0) - \rho_{DV}^c \frac{\partial \lambda_{DV}(k_E, m_D, 0)}{\partial \rho_{DV}^c}. \quad (41)$$

Here,

$$\lambda_{DV}(k_E, m_D^*, \rho_{DV}^c) = \frac{4(k_E^2 E_\omega + m_D^{*2})z^2 + k_E^2(4E_\omega - 3)}{2m_D^* E_\omega k_E^2 (\sqrt{z^2(z^2 + 1)} + z^2)} - \frac{1}{m_D^*}. \quad (42)$$



The counter term in the Lagrangian density, responsible for the renormalization of the vector density, has the form  $\delta\mathcal{L}_v = C_3(\partial_\mu - ih_\omega\omega_\mu)\varphi^*(\partial_\mu + ih_\omega\omega_\mu)\varphi$ . The structure of the counter terms shows that the model is renormalizable to one loop.

Despite the fact that a dibaryon Bose condensate does not exist in ordinary nuclei, dibaryons affect properties of nuclear matter and nuclei through a Casimir effect described by Eq.(30). This effect is analysed in Ref. [30]. When  $x \rightarrow 1$   $\zeta(x) = O((1-x)^4)$ , so the vacuum contributions to the scalar density of nucleons and dibaryons, which have opposite signs, are comparable for  $4g_\sigma^4/m_N \approx h_\sigma^4/m_D$ . Dibaryon effects become large for  $h_\sigma/(2g_\sigma) \approx 0.5(4m_D/m_N)^{1/4} \approx 0.84$ . Explicit calculation [30] shows that the basic properties of nuclear matter at saturation density can be reproduced for  $h_\sigma/(2g_\sigma) < 0.8$ , and that the results are not very sensitive to the dibaryon mass. The parameter sets of the RHA model for the H-particle, and the d', and d<sub>1</sub>-dibaryons are shown in Table 1. The meson masses  $m_\sigma = 520$  MeV and  $m_\omega = 783$  MeV and the equilibrium Fermi wave number  $k_F = 1.3$  fm<sup>-1</sup> are the same as in RHA without dibaryons [25, 26].

The effective nucleon and dibaryon masses, which are obtained as solutions of the self-consistency condition (8) are shown in Fig.1a. In Fig.1 (b-d) the scalar and vector densities for nucleons and dibaryons, and the EOS of HNMD are plotted versus the total baryon number density  $\rho_{TV}$  for  $h_\sigma/(2g_\sigma) = 0.6$  and  $h_\omega/(2g_\omega) = 0.8$ . The numerical results are given for  $\gamma_N = 4$  (at low density we start from nuclear matter) and for the d' dibaryon mass  $m_D = 2060$  MeV. At densities  $\rho_{TV} < \rho_1$ , the chemical potential of dibaryons is larger than twice the chemical potentials of nucleons,  $2\mu_N < \mu_D$ , so we have normal nuclear matter. In the interval  $\rho_1 < \rho_{TV} < \rho_2$ , nucleons and dibaryons are in chemical equilibrium, i.e.  $2\mu_N = \mu_D$ , and the density of nucleons decreases with increasing total baryon number density. In the interval  $\rho_2 < \rho_{TV} < \rho_3$ , the dibaryon chemical potential is inside of the energy gap for nucleons,  $2(-m_N^* + g_\omega\omega_c) < \mu_D < 2(m_N^* + g_\omega\omega_c)$ , and nuclear matter consists of dibaryons only. At densities  $\rho_3 < \rho_{TV}$ , antinucleons come into chemical equilibrium with dibaryons, the relation  $2(-\mu_N^* + g_\omega\omega_c) = \mu_D$  holds true. The density of antinucleons increases with the total baryon number density. The numerical values for the densities  $\rho_1$ ,  $\rho_2$ , and  $\rho_3$  of the nuclear and neutron matter are given in Table 1.

The energy density and pressure are calculated from

$$\varepsilon = \int \mu_N d\rho_{TV} \quad \text{and} \quad p = \int \rho_{TV} d\mu_N. \quad (43)$$

In the interval  $\rho_2 < \rho_{TV} < \rho_3$ , one should put  $2\mu_N = \mu_D$  by definition.

## 5 Conclusion

In an interacting Bose gas, a fraction of bosons is not in the condensate [21]-[23]. Non-condensate dibaryons appear in a one-loop calculation of quantum hadrodynamics. This approximation corresponds to the relativistic Hartree approximation for normal nuclear matter. We have constructed the Green's functions of heterophase nucleon-dibaryon matter and have calculated the contribution of non-condensate dibaryons to the scalar and vector densities and the equation of state. This contribution increases rapidly with the total baryon number density and becomes dominant as compared to the contribution of condensate dibaryons. In mean field theory, the effective nucleon mass vanishes with increasing density [13]-[15]. In relativistic Hartree approximation, the effective nucleon mass is finite and positive. In a loop expansion of quantum hadrodynamics, the relativistic Hartree approximation is the lowest approximation sufficient to account for the presence of non-condensate dibaryons in heterophase nucleon-dibaryon matter, while providing consistent solutions at high densities.

The present one-loop calculation allows (i) to consider the structure of neutron stars with dibaryons in the interiors without restrictions to the total density and (ii) to construct the phase diagram for the phase transition of normal nuclear matter to dibaryon matter and quark matter without restrictions to the densities and temperatures.

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Table 1

Dibaryon	Mass (GeV)	$h_\sigma/(2g_\sigma)$	$g_\sigma$	$g_\omega$	$\rho_1/\rho_0$	$\rho_2/\rho_0$	$\rho_3/\rho_0$	$\gamma_N$
H	2.22	0.6	8.5277	10.4393	3.62	11	33	2
					4.86			4
d'	2.06	0.6	8.5438	10.4687	2.48	7.9	30	2
					3.52			4
d <sub>1</sub>	1.92	0.6	8.5606	10.4993	1.31	4.6	29	2
					2.23			4

## Figure captions

**Fig.1.** (a) The effective nucleon and dibaryon masses versus the total baryon number density. (b) The scalar density of nucleons of Eq.(25): the contribution of (i) nucleons in the Fermi sphere ( $\rho_{NS}^c$ , solid line), (ii) the negative Dirac sea ( $\rho'_{NS}$ , dotted line). The individual contributions to the scalar density of dibaryons according to Eq.(31): (i) dibaryons in the condensate ( $2m_D^*\rho_{DS}^c$ , solid line), (ii) the positive contribution of the zero-point dibaryon fluctuations ( $2m_D^*\rho'_{DS}$ , dotted line), and (iii) the term  $2m_D^*\rho''_{DS}$  (dashed line) describing the contribution of dibaryons not in the condensate. The densities are measured in units of the saturation density  $\rho_0$  of nuclear matter. Note the different dimensions of the scalar nucleon ( $[fm^{-3}]$ ) and dibaryon densities ( $[fm^{-2}]$ ). (c) The fraction of nucleons ( $\rho_{NV}/\rho_{TV}$ ), and dibaryons ( $\rho_{DV}^c/\rho_{TV}$ ) in the condensate (solid lines). The fraction of dibaryons not in the condensate ( $\rho''_{DV}/\rho_{TV}$ ) (dashed line). (d) The energy density and pressure of heterophase nucleon-dibaryon matter (dashed lines) compared to the RHA without dibaryons (solid lines) versus the total baryon number density. At densities  $\rho_{TV} < \rho_1$  normal nuclear matter ( $\gamma_N = 4$ ) is stable. In the interval  $\rho_1 < \rho_{TV} < \rho_2$ , nucleons and dibaryons are in chemical equilibrium. In the interval  $\rho_2 < \rho_{TV} < \rho_3$ , only dibaryons are present in nuclear matter. At densities  $\rho_3 < \rho_{TV}$  dibaryons are in chemical equilibrium with antinucleons.